ERRATUM TO EIGENVALUE PINCHING FOR RIEMANNIAN VECTOR BUNDLES

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Lemma 3.1 of [3] is incorrect. The correct statement is the following.

Lemma 0.1. *Let u be a function on a compact manifold with* $\mathrm{Ric} \geq -k^2$, diam $<$ D *such that* $\Delta u \leq \alpha u + \beta$. *Then there exist* $K = K(n, k, D, \alpha, \beta)$ *and* r, *with* $e^{-(n-2)\ln(2)/2} < r \leq 1$, *such that* $\|u\|_\infty \leq (K\|u\|_2)^r$.

Proof. From [4, eq. (3.8)],

$$\|u\|^{2p}_{\frac{2np}{n-2}} \leq C_S \frac{p^2}{2p-1}\left(\alpha\|u\|^{2p}_{2p} + \beta\|u\|^{2p-1}_{2p}\right) + \|u\|^{2p}_{2p}, \tag{1}$$

Where $C_S = C(n, k, D)$.

Let $p_i = \left(\frac{n}{n-2}\right)^i$. Note that for any $A_i, B_i > 0$ we have that $(A_i+B_i)\min\{\|u\|^{2p_i}_{2p_i}, \|u\|^{2p_i-1}_{2p_i}\} \leq A_i\|u\|^{2p_i}_{2p_i} + B_i\|u\|^{2p_i-1}_{2p_i} \leq (A_i+B_i)\max\{\|u\|^{2p_i}_{2p_i}, \|u\|^{2p_i-1}_{2p_i}\}$. In particular for for each i we can find r_i such that $2p_i - 1 < r_i < 2p_i$ and

$$\left(\alpha C_S \frac{p_i^2}{2p_i-1} + 1\right)\|u\|^{2p_i}_{2p_i} + \left(\beta C_S \frac{p_i^2}{2p_i-1}\right)\|u\|^{2p_i-1}_{2p_i} = \left((\alpha+\beta) C_S \frac{p_i^2}{2p_i-1} + 1\right)\|u\|^{r_i}_{2p_i} \tag{2}$$

Therefore,

$$\|u\|_{\frac{2np_i}{n-2}} \leq \left((\alpha+\beta) C_S \frac{p_i^2}{2p_i-1} + 1\right)^{\frac{1}{2p_i}} \|u\|^{\frac{r_i}{2p_i}}_{2p_i} \tag{3}$$

Iterating gives

$$\|u\|_\infty \leq \left(\prod_{i=1}^{\infty}\left((\alpha+\beta) C_S \frac{p_i^2}{2p_i-1} + 1\right)^{\frac{1}{2p_i}} \|u\|_2^{\frac{r_0}{2}}\right)^{\prod_{i=1}^{\infty}(r_i/2p_i)} \tag{4}$$

The first product above is well-known to converge to a finite value. Let $r = \prod_{i=0}^{\infty}(r_i/2p_i)$. Since $r_i \leq 2p_i$ we clearly have $r \leq 1$. To show that $r > 0$, we use that $r_i \geq 2p_i - 1$ which gives $r \geq \prod_{i=0}^{\infty}(1 - \frac{1}{2p_i}) = \prod_{i=0}^{\infty}(1 - \frac{1}{2}(\frac{n-2}{n})^i)$. Taking a logarithm and estimating by a geometric series shows that this product lies in the interval $[e^{-(n-2)\ln 2/2}, e^{-(n-2)/4}]$. ∎

This gives the following which is sufficient for the applications in [3], [4]. Let E be a Riemannian vector bundle over M with $|R^E|, |\nabla R^E| < K$, $\mathrm{Ric}_M \geq -k^2$, diam(M) $< D$. Then for L^2-orthonormal eigensections $S_1, ..., S_m$ of E with eigenvalues $\lambda_1, ..., \lambda_m$, the formulas

$$\|\langle S_i, S_j\rangle - \delta_{ij}\|_\infty \leq \tau(\lambda_m | n, k, K, R)$$

$$\|\nabla S_i\|_\infty \leq \tau(\lambda_m | n, k, K, R)$$

hold, where $\tau \to 0$ as $\lambda_m \to 0$.

See [1], [2] for arguments which lead to similar results without requiring a uniform bound on ∇R^E. We would like to thank W. Ballman for pointing out this error to us.

References

[1] W. Ballman, J. Bruning, and G. Carron, Eigenvalues and holonomy, preprint.

[2] B. Colbois, P. Ghanaat, and E. Ruh, Curvature and a spectral characterization of nilmanifolds, preprint.

[3] P. Petersen and C. Sprouse, Eigenvalue pinching for Riemannian vector bundles, J. Reine Angew. Math. **511** (1999), 73-86.

[4] P. Petersen and C. Sprouse, Eigenvalue pinching on p-forms, Proceedings of the Fifth Pacific Rim Geometry Conference, Tohoku Math. Pub. **20** (2001), 139-145.